Respond in English or Español.

1. True/False (1 pt each)

   (a) Gain variations can provide fundamental limitations to your sensitivity. __T__

   (b) Unlike a perfect parabolic reflector, the transmitting power pattern of a dipole can differ from the receiving power pattern. __F__

   (c) Although we initially measure temperatures, these are quickly converted into voltages. __F__

   (d) A dipole antenna requires two parts and must be "grounded" to work. __F__

   (e) Adding metal (capacitance) to the end(s) of a dipole or monopole antenna will increase their radiation efficiency. __T__

   (f) Monopoles behave like dipoles by using a reference plane. __T__

   (g) Blackbody radiation is often polarized by the emitting substance. __F__

   (h) A $T_\mathrm{B} = 10^4$ K source covering 1% of the beam solid angle will add 100 K to the antenna temperature. __T__

   (i) Bolometers rely on redundancy to observe faint sources. __T__

   (j) Impedance dictates how we couple energy into the antenna structure. __T__

   (k) A synchrotron radiation spectrum is continuous. __F__

   (l) Dust contributes strongly to atmospheric opacity. __F__

   (m) The Ruze equation depends on wavelength and diameter of the telescope. __F__

   (n) Radio emission is primarily produced by electrons. __T__

   (o) Interferometers work on the principle of compensating for geometric and atmospheric delays. __T__

   (p) The far-field limit of a source from a reflecter is proportional to the frequency. __T__

   (q) The higher the gain, the narrower the antenna main beam. __T__

   (r) Area is measure of directionality of receiver (compared to 4π). __T__

   (s) Waveguides and dipoles two distinct types of antennas. __F__

   (t) Flux density is dependent on distance. __T__

   (u) The Rayleigh-Jeans Law and Wien's Law are both approximations of the Planck function. __T__

   (v) As the distance between antennae increases, can be more difficult to measure "fringes" accurately. __T__

   (w) The more antennae involved in an interferometer, the rounder the beam shape, no matter what configuration they are in. __T__

   (x) When calibrating interferometer data, one must calibrate the amplitude, bandpass, and phase before one can make a good image. __T__

   (y) Bolometers were invented after the CCD camera, so that radio telescopes could act like optical ones. __F__

2. At full capacity, the Atacama Large Millimeter Array (ALMA) will consist of 66 antennae ($50 \times 12\,\mathrm{m} + 16 \times 7\,\mathrm{m}$) that sample baselines from 200 m to 16 km and cover frequencies from 84 GHz to 950 GHz with a bandwidth of 16 GHz. Although ALMA is extremely sensitive, it is *NOT* considered a good survey instrument (i.e., for studying large regions on the sky). Why? To answer this, let us consider the following.

   (a) (4pts) Estimate the system temperature T_{sys} at zenith and 30 elevation, assuming the telescopes are roughly background-limited (i.e., sky-dominated) and have atmospheric and ambient temperatures at Chajnantor of $\approx 260\,\mathrm{K}$ each. Atmospheric skydip measurements at the high-site find median broadband opacities to be $\tau_{230\mathrm{GHz}} \sim 0.1$, $\tau_{650\mathrm{GHz}} \sim 0.9$, and $\tau_{850\mathrm{GHz}} \sim 2.5$. Assume the telescopes have a reasonably high efficiency of $\eta \sim 0.9$.

Solution: Background-limited means $T_{\mathrm{atmos}} + T_{\mathrm{ambient}}$ dominate T_{sys}. Since $\tau_{\mathrm{A}} = \tau_{\mathrm{z}} \sec(z)$, we have

$$\begin{aligned}
\tau_{30\,\mathrm{deg}} &= \tau_{0\,\mathrm{deg}} \sec(60^\circ) = 2 \times \tau_{0\ \mathrm{deg}} \\
T_{\mathrm{sys}} &= T_{\mathrm{receiver}} + \eta T_{\mathrm{atmos}}[1 - e^{-\tau}] + (1-\eta) T_{\mathrm{ambient}} \approx T_{\mathrm{ambient}} - \eta T_{\mathrm{atmos}} e^{-\tau}
\end{aligned}$$

See table below for calculated values

   (b) (4pts) Given the above T_{sys}, estimate the time required for a detection in one resolution element on the sky for 0.5 mJy.

Solution: Use interferometer radiometer equation. Since we have two different telescope sizes, we will estimate the "average" area as $A_{\mathrm{avg}} = \sqrt{[50 \times (12)^2 + 16 \times (7)^2]/66}$ =11 m. Thus

$$\begin{aligned}
\sigma_{\mathrm{S}} &= \frac{2kT_{\mathrm{sys}}}{A_{\mathrm{eff}}[N(N-1)\Delta\nu t]^{(1/2)}} \\
t &= \frac{(2kT_{\mathrm{sys}})^2}{[\sigma_{\mathrm{S}} \eta A_{\mathrm{avg}}]^2 N(N-1)\Delta\nu} \\
&= \frac{(2 \times 1.38 \times 10^{-16}\,\mathrm{erg/K}\ \ T_{\mathrm{sys}})^2}{[0.5 \times 10^{-26}\,\mathrm{erg/s/cm^2/Hz} \times 0.9 \times \pi(1100\ \mathrm{cm}/2)^2]^2 \times 66(65) \times 16 \times 10^9\,\mathrm{Hz}} \\
&= 6.07 \times 10^{-5}\ T_{\mathrm{sys}}^2\ \mathrm{s}
\end{aligned}$$

   (c) (6pts) Given that the primary beam of ALMA at 100 GHz is 56″, how long would it take to map ∼1 sq. deg of sky at 230, 650, and 850 GHz? This rate per sq. deg is often called a 'figure of merit' survey speed.

Solution:

$$\begin{aligned}
\text{FOV} &\propto \frac{4\pi A_{eff}}{\lambda^2} \\
\text{FOV}_\nu &= \text{FOV}_{100\,\text{GHz}} \times \left(\frac{100\,\text{GHz}}{\nu}\right)^2 \\
\text{surveyspeed} &= t\,\frac{1\text{deg}^2}{\text{FOV}_\nu/(3600)^2} \\
&= 0.319(T_{\text{sys}})^2\left(\frac{\nu}{100\,\text{GHz}}\right)^2
\end{aligned}$$

ν	τ_{0°	$T_{\text{sys},0^\circ}$	t (s)	survey speed (s)
230 GHz	0.1	48.3 K	0.14	2.97×10^3
650 GHz	0.9	164.9 K	1.65	3.12×10^5
850 GHz	2.5	240.8 K	3.52	1.18×10^6
ν	τ_{60°	$T_{\text{sys},60^\circ}$	t (s)	survey speed (s)
230 GHz	0.2	68.4 K	0.28	5.97×10^3
650 GHz	1.8	221.3 K	2.97	5.62×10^5
850 GHz	5.0	258.4 K	4.05	1.36×10^6

Thus it would take 1 hr/deg at 230 GHz and 13 days/deg at 850 GHz. This shows how ineffective ALMA will be for making a survey of any realistic size. It would literally years even at 230 GHz!

(d) (10pts) Now consider a single 30m submm telescope located at either the South Pole (90° S) or Chajnantor (22° S, along with ALMA), equipped with a sensitive multi-beam bolometer (100 GHz bandwidth). Initial FOV would be 20′ and expandable to 0.9° by 2025. Note at the South Pole, $T_{\text{atmos}} \approx 200\,\text{K}$ and $\tau_{350\mu\text{m}} \sim 2.0$. How does a telescope like this at either site compare in terms of survey speed? Also, how many individual bolometers will be required at the three primary bands?

Solution:

$$
\begin{aligned}
t &= \frac{(2kT_{\rm sys})^2}{[\sigma_{\rm S}\eta A_{\rm avg}]^2\Delta\nu} \\
t &= 7.53\times10^{-4}\,(T_{\rm sys})^2 \\
\text{surveyspeed} &= t\,\frac{1\text{deg}^2}{\text{FOV}_\nu/(3600)^2} \\
\text{initialspeed} &= 8.63\times10^{-3}(T_{\rm sys})^2\left(\frac{\nu}{100\,\text{GHz}}\right)^2 \\
\text{ultimatespeed} &= 1.18\times10^{-3}(T_{\rm sys})^2\left(\frac{\nu}{100\,\text{GHz}}\right)^2
\end{aligned}
$$

Chajnantor

ν	τ_{0°	$T_{\rm sys,0^\circ}$	t (s)	initial survey speed (s)	ultimate survey speed (s)
230GHz	0.1	48.3 K	97	4.46×10^3	6.11×10^2
650GHz	0.9	164.9 K	1130	4.67×10^5	6.41×10^4
850GHz	2.5	240.8 K	2410	1.77×10^6	2.43×10^5
ν	τ_{60°	$T_{\rm sys,60^\circ}$	t (s)	initial survey speed (s)	ultimate survey speed (s)
230GHz	0.2	68.4 K	195	8.94×10^3	1.22×10^3
650GHz	1.8	221.3 K	2040	8.42×10^5	1.15×10^5
850GHz	5.0	258.4 K	2780	2.04×10^6	2.80×10^5

South Pole

ν	τ_{0°	$T_{\rm sys,0^\circ}$	t (s)	initial survey speed (s)	ultimate survey speed (s)
850GHz	2.0	175.6 K	23.2	1.70×10^4	2.33×10^3
ν	τ_{60°	$T_{\rm sys,60^\circ}$	t (s)	initial survey speed (s)	ultimate survey speed (s)
850GHz	4.0	196.7 K	29.1	2.14×10^4	2.92×10^3

Because of background and depth, the Chajnantor single dish is initially worse. However, with a large enough FOV it can compensate for the (N(N-1)) factor to be a factor of 5 faster. The lower background at the South Pole demonstrates a huge advantage for surveys. A 30 m would have resolutions of 9.0″, 3.2″, 2.1″ at 230 GHz, 650 GHz, and 950 GHz, and thus we would need bolometer arrays of order 17904, 143000, and 305463 initially and 130525, 1042477, and 2226829 eventually. 2 megapixels more or less, which is becoming feasible these days.

(e) **You may earn some extra credit** by debating the pros and cons of the telescope being located at the South Pole vs. Chajnantor (1 pt for each valid point you make, but -1 pts for each incorrect one, so debate wisely).

Solution: The South Pole has a (*) lower $T_{\rm atmos}$, which means (*) less water vapor (so cold that it freezes out), and yields a much deeper sensitivities as shown above. However, the harsh conditions make it (*) much harder to access, (*) more expensive, and thus (*) potentially significantly less efficient versus Chajnantor. Because a South Pole telescope would lie at -90° , it has (*) long nights for a significant fraction of the year and hence less thermal changes. However, this location also means that (*) it can only effectively see observe to perhaps ∼-30° on the sky, so it effectively only accesses a third of the sky that can be seen from Chajnantor. (*) Chajnantor's much higher elevation helps get above a good portion of the atmosphere contaminants.

3. While visiting the 100 m Green Bank Telescope (it was on the way to an annual hillbilly festival you were attending), your tourguide accidently leaves you alone in the control room. You decide to have some fun and slew toward the nearest Messier object you can find, M1 (∼2 kpc away), and start taking data.

   (a) (5pts) You discover that this source has a flux of 6 Jy at 850 MHz and 27 Jy at 1.4 GHz. What is the spectral index of the source. It is unresolved by the GBT, so what is its minimum brightness temperature with the information you have. Could it be an HII region or a typical synchrotron source based on this information alone?

Solution:

$$
\begin{aligned}
\alpha &= -d\log S_\nu / d\log\nu = -[\log(27) - \log(6)]/[\log(1.40) - \log(0.85)] = -3.01 \\
S_{1.4\rm GHz} &= B_{1.4\,\rm GHz}\Omega = 27\,\rm Jy = 2.7\times10^{-22}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1} \\
\Omega &< \pi\left(\frac{1.2\lambda}{2D}\right)^2 = \pi\left(\frac{1.2\times21.4\,\rm cm}{100\,m\times2}\right)^2 = 1.63\times10^{-5}\,\rm sr \\
B_{8.5\rm GHz} &= \frac{2kT_{\rm B}\nu^2}{c^2} \\
T_{\rm B} &> \frac{(3\times10^{10}\,\rm cm)^2\times2.7\times10^{-22}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}{2\times(1.38\times10^{-16}\,\rm erg/K)\times(1.4\times10^{9}\,Hz)^2\times(1.63\times10^{-5}\,sr)} = 27.6\,\rm K
\end{aligned}
$$

From the spectral index, it seems unlikely to be a thermal source (HII region), but the temperature is compatible even when extrapolated into the mm-bands. It is also unresolved, however, so the source size could be much smaller (leading to a much larger temperature).

   (b) (10pts) Looking through the data, you discover what you think are 12 ns pulses every 0.15121 s. It appears to be a pulsar! Intrigued by your discovery, you convince the staff astronomer to make another observation the following day. The new data show that the period between days has lengthened by 18.7 nanoseconds per day. Now you can estimate the pulsars total rotational energy and spin-down luminosity. How does the radio luminosity compare to this?

Solution: Note that $\dot{P} = \dfrac{18.7\,\text{ns}}{1\,\text{day}} = 2.16 \times 10^{-13}$. So

$$\begin{aligned}
E_{\text{rot}} &= \frac{2\pi^2 I}{P^2} = \frac{4\pi^2 M R^2}{5P^2} = \frac{4\pi^2 \times 1.4 \times 2 \times 10^{33}\text{g} \times (10^6\,\text{cm})^2}{5 \times (0.15121\,\text{s})^2} = 9.68 \times 10^{47}\,\text{erg} \\
L_{\text{spindown}} &= \frac{dE_{\text{rot}}}{dt} = \frac{-4\pi^2 I \dot{P}}{P^3} = \frac{-2E_{\text{rot}}\dot{P}}{P} = \frac{-2 \times 9.68 \times 10^{47} \times 2.16 \times 10^{-13}}{0.15121} \\
&= -2.72 \times 10^{36}\,\text{erg/s} \\
L_{\text{radio}} &= 4\pi d^2 \int_{10\,\text{MHz}}^{100\,\text{GHz}} S_\nu d\nu = 4\pi (6.2 \times 10^{21}\,\text{cm})^2 \int_{10^7}^{10^{11}} 27\,\text{Jy} \left(\frac{\nu}{1.4 \times 10^9}\right)^{-3} d\nu \\
&= 3.58 \times 10^{50} \left[\frac{\nu^{-2}}{-2}\right]_{10^7}^{10^{11}} = 1.79 \times 10^{36}\,\text{erg/s}
\end{aligned}$$

They are surprisingly comparible, such that the radio luminosity could be driving the spindown entirely here.

(c) (3pts) Is your pulsar young enough to bother looking for a companion supernova remnant? How does it compare to other objects on the P$\dot{P}$ diagram?

Solution:

$$t_{\text{age}} = \frac{P}{2\dot{P}} = \frac{0.15121\,\text{s}}{2 \times 2.16 \times 10^{-13}} = 11091\,\text{yrs}$$

So yes, we would potentially expect to have a faint SNR companion. In the P$\dot{P}$ diagram, it lies near young pulsars like Vela and Crab.

(d) (4pts) Estimate the dispersion measure of your pulsar through the ISM and its affect on the pulse width. If there is a 10% error in the delay times, at what frequencies will the pulse become lost in the noise?

Solution: We have $\Delta t_{\text{pulse}} = 12\,\text{ns}$ at 1.4 GHz, which is a measure of dispersion smearing (derivative of dispersion delay, t_{delay}) and yields DM (and n_{e}). Let's assume we have 1 Mhz channels for $d\nu$ below. Then want to solve for ν that will give $t_{\text{smear}} = \Delta t_{\text{delay}}$ roughly equal to t_{pulse} (well, 90% of t_{pulse} to account for 10% error).

$$\begin{aligned}
t_{\text{delay}} &= 4.149 \times 10^3 \left(\frac{DM}{\text{pc}\,\text{cm}^{-3}}\right) \left(\frac{\nu}{\text{MHz}}\right)^{-2} \\
\text{so } \Delta t_{\text{delay}} &= -8.3 \times 10^3 \left(\frac{DM}{\text{pc}\,\text{cm}^{-3}}\right) \left(\frac{\nu}{\text{MHz}}\right)^{-3} d\nu \\
\text{solve for } \frac{DM}{\text{pc}\,\text{cm}^{-3}} &= \frac{\Delta t_{\text{delay}} (\nu/1\,\text{MHz})^3}{d\nu} = \frac{12\,\text{ns}\ (1.4\,\text{GHz}/1\,\text{MHz})^3}{10^6\,\text{Hz}} = 3.29 \times 10^{-5} \\
\nu_{\text{lost}} &< \left[8.3 \times 10^3 \left(\frac{DM}{\text{pc}\,\text{cm}^{-3}}\right) \times (0.9 t_{\text{pulse}})^{-1} \times d\nu\right]^{1/3} = 33.4\,\text{MHz}
\end{aligned}$$

4. (15pts) Estimate what the typical thermal continuum flux density at 100GHz would be for an earthlike planet orbiting at 1AU from a solar-type located 20 pc away. Assume the Earth is a blackbody that absorbs and re-radiates all incident optical light from its star. What minimum angular resolution would be needed to spatially resolve this planet from its more dominant star? How large of a radio telescope would we need to observe this below 1 mm?

Solution: This is nearly identical to that of Mars in tarea 1. Calculate Sun luminosity, determine flux intercepted by Earth, determine temperature on Earth, determine luminosity and hence flux at 20 pc.

$$L_\odot = 3.9 \times 10^{33} \text{ erg/s}$$

Neglecting potential differential heating issues, we have

$$\begin{aligned}
\frac{L_{\text{planet}}}{A_{\text{planet}}} &= \frac{L_\odot}{A_{\text{sphere of 1 AU}}} \\
4\pi R^2_{\text{planet}} \sigma T^4_{\text{planet}} &= \frac{\pi R^2_{\text{planet}}}{4\pi d^2} \times L_\odot \\
T^4_{\text{planet}} &= \frac{R^2_\odot}{4d^2} T^4_\odot \\
T_{\text{planet}} &\approx 278.8 \text{ K}
\end{aligned}$$

The planet size and angular separation between the star and planet are

$$\begin{aligned}
\theta_{\text{sep}} &= \frac{1\,\text{AU}}{20\,\text{pc}} = \frac{1.50 \times 10^{13}}{20 \times 3.086 \times 10^{18}} = 2.43 \times 10^{-7} = 0.05'' \\
\theta_{\text{planet}} &= \frac{6.3 \times 10^8\,\text{cm}}{20\,\text{pc} \times 3.086 \times 10^{18}} = 1.02 \times 10^{-11} = 2.1 \times 10^{-6\prime\prime}
\end{aligned}$$

To resolve the planet at 1 mm = 300 GHz (this is the best wavelength for resolving since its depends on λ^{-2}), we simply want a telescope that can separate the star and planet (at least half the separation). If we were worried about detecting the planet, then we would have to further deduce the dependence on the star's flux at some beam fraction as well.

$$\begin{aligned}
\theta_{\text{sep}} &= 1.2\,\frac{\lambda}{D} \\
D &> 1.2\,\frac{0.001\,\text{m}}{2.43 \times 10^{-7}/2} \text{radians} \;= 2.5\,\text{km}
\end{aligned}$$

Now we can use this size to find the flux density.

$$\begin{aligned}
S_{\text{planet}} &= B_\nu \Omega = \frac{2kT\nu^2}{c^2} \times \pi \left(\frac{\theta_{\text{planet}}}{2}\right)^2 \\
&= \frac{2 \times 1.38 \times 10^{-23} \times 278.8 \times (300 \times 10^9)^2}{(3 \times 10^8)^2} \times \pi \left(\frac{1.02 \times 10^{-11}}{2}\right)^2 \\
&= 6.29 \times 10^{-37} \text{ W m}^{-2} \text{ Hz} = 6.34 \times 10^{11} \text{ Jy}
\end{aligned}$$