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Transit-timing variations

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ABSTRACT
The total number of exoplanets detected is rising constantly with time, but the number of jovian-like planets is huge compared to earth-sized discovered planets. In the present work we sumarize results and discoveries of a relatively new method of exoplanet detection called transit-timing variations. The analysis is described both quantitatively and qualitatively, along with a discussion on some of the most recent discoveries and suggestions made by this method.

Key words: extrasolar planets – methods: transit timing variations.

1 INTRODUCTION

The recent detection of the 107th transiting exoplanet (Anderson et al, 2010) has expanded the posibility of detailed study to $\sim 21\%$ of the total exoplanets discovered so far[1]. From atmosphere composition detection to different orbital and physical parameters, the transit method has proven to be one of the most interesting methods of exoplanetary study (see Winn (2010) for a review). So far, 81% of 114 exoplanets has jovian or greater radii, whereas 75% of the 505 currently known exoplanets has jovian or greater masses (from $0.5M_{♃}$ to $20M_{♃}$). Although these results may appear discouraging at first glance for earth-like planet detection, the information that these kind of planets can provide is unique.

Suppose we observe a transit a number of j times, where $0 \leqslant j \leqslant N$ and N is the total number of observed transits (or, if you like, take j as the "day of the j-th transit". Here j and t are interchangeable.). At time $t(j)$, we observe the minimum flux of the jth transit, so the period P of the transiting exoplanet can be found by applying a linear regression to the function,

$$t(j) = jP + t(0)$$

If periodic variations arise in this function we identify a transit-timing variation. According to this, it is defined the transit-timing variation signal (TTV signal) as,

$$\delta t(j) = t(j) - jP$$

Note that $\delta t(j)$ measures the deviation of $t(j)$ from a strictly periodic (i.e. single-frequency) signal. Identifiying different types of variations is important because different signals will appear for different physical processes. Figure 1 shows a synthetically created TTV signal for illustration. The goal of the present work will be to investigate what information the TTV signal can give us and how to interpret it.

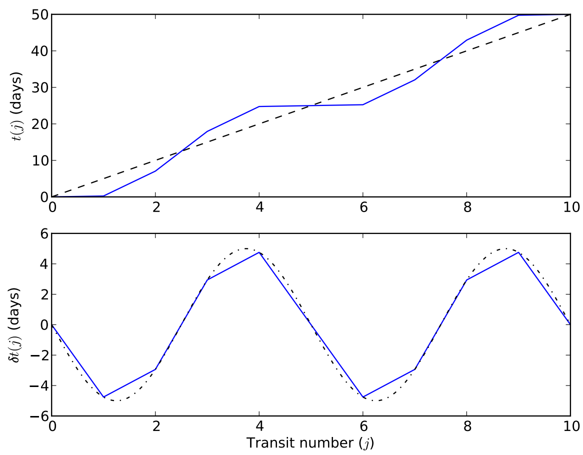

Figure 1. Illustrations of synthetically created transit curves, where we took $t(0) = 0$ for simplicity. *Up:* Time of transit observation v/s number of transit. The dashed line represents the linear regression to the function in order to get the period P of the transit, while the solid curve represents the hyphotetical transit times. *Down:* TTV Signal for the data. A clear periodic variation can be seen with a 5-day amplitude.

2 TRANSIT-TIMING VARIATIONS: PLANET INTERACTIONS

In theory, one could directly make a N-body integration of a planetary system and compare these results with the ob-

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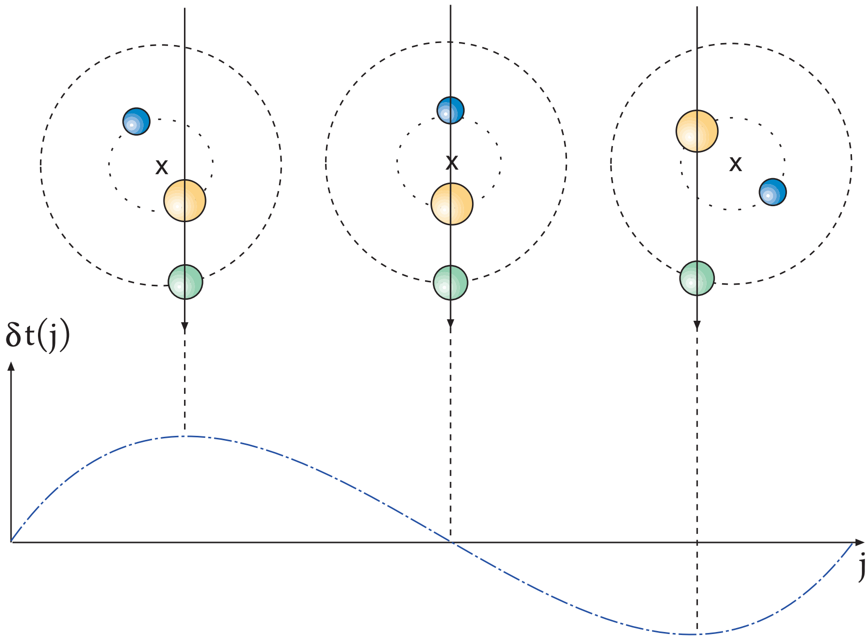

Figure 2. Illustrations of the case of non-interacting planets. The yellow, blue and green spheres represent the central star and the central and transiting planet, respectively. Note that our TTV signal has now a physical meaning: the variation is made entirely by the relative motion of the central binary system.

tained TTV signal. However, the parameter space for this kind of method is extremely large, considering the distinct possibilities of orbital parameters and masses involved in these calculations, which is therefore extremely CPU expensive. For example, as explained by Nesvorný & Morbidelli (2008), in the simplest case of near-zero eccentricities and coplanar orbits a total of 10^{10} planetary systems are needed to track the TTV signal obtained from measurements. It is therefore useful to constrain certain parameters given a TTV signal, and from there constrain the parameter space not only to get less CPU expensive, but also to have a clear physical picture of what's creating the variation on the transit time. Here we sumarize some of the typical processes described in two of the classical papers in transit timing variations by Agol et al. (2005) and Holman & Murray (2005).

2.1 Time-variations by non-interacting planets

At first glance it can sound odd that non-interacting planets can cause transit-timing variations, but the fact that we observe the entire orbiting planetary system is a privilege because we can observe the motion of the planets and the central star around the center of mass. Therefore, if we observe a transiting planet that is relatively far away from the central star, we can observe perturbations in the transit time if there's other planets orbiting the central star closer than our transiting planet, if they are massive and/or close enough to form a central binary system.

Consider a central binary system formed by the central star and a close-orbiting planet, hereafter the central planet and a transiting planet that is far enough from the center of mass in order to not to be perturbed by the inner binary orbiting bodies. If the central planet is massive and/or close enough to the host star, the motion of the star relative to the center of mass will be seen by a distant observer that observes a transit of the outer planet, because the line of sight of the transit will be shifted as a function of the forms of the orbits. Figure 2 illustrates our point in the case of two planets orbiting a star along with the respective TTV signal.

The exact solution of this problem can be obtained directly in this case by solving the Kepler problem for each planet separately, because we aren't considering perturbations between planets (i.e. the central binary system can be solved as a two body problem considering the central planet and the star and the outer transiting planet can be solved as orbiting the reduced mass of the inner binary system). Therefore, the computational problem in this case is relatively easy (see, for example, Murray & Dermott, 1999). For illustration, the TTV signal for the circular case is obtained by taking the shift of the inner star from the center of mass of the inner binary system produced by the central planet (x_s), and dividing it by the relative velocity between the star ($v_\star$) and the outer (transiting) planet (v_t), where if we consider that the velocity of the star is considerably small than the velocity of the outer planet ($v_\star << v_t$), we get:

$$\delta t(j) \approx \frac{x_s}{v_t} \approx -\left(\frac{P_t}{2\pi}\right)\left(\frac{a_c}{a_t}\right)\left(\frac{m_c}{M_\star}\right)\sin\left(\frac{2\pi[jP_t - t(0)]}{P_c}\right)$$

Where a_c, P_c and a_t,P_t represent the radii and period of the inner and outer (transiting) planet orbits with respect to the center of mass system, m_c is the mass of the central planet and $M_\star$ is the mass of the central star. An important observable is the r.m.s. intensity of the TTV signal, which is given by:

$$< \delta t >_{rms} = \sqrt{\frac{1}{\tau}\int_0^\tau \delta t(j)^2 \, dt} = \frac{1}{\sqrt{2}}\frac{P_t a_c m_c}{2\pi a_t M_\star}$$

On the worst case scenario, we can observe transit timings

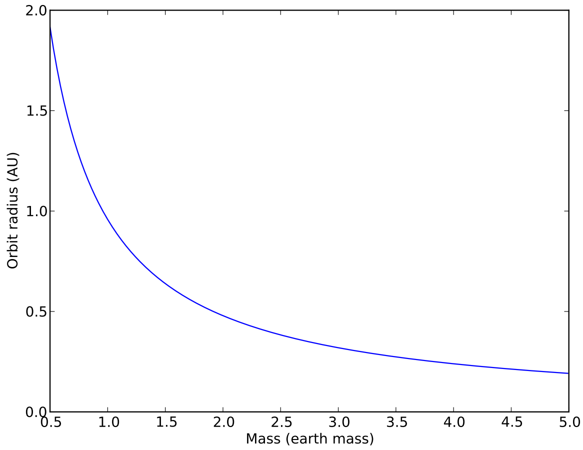

Figure 3. Curve of possible a_c and m_c values obtainable from a transit variation of a planet with a period of 11 years at an orbital radius of Jupiter, from a central star like the sun.

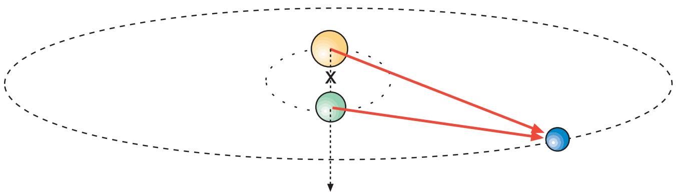

Figure 4. Illustration of the perturbation made by an outer planet on the inner binary system just at the time of transit. Again, the yellow sphere represents the central star and the green and blue spheres represent the transiting and perturbing planets, respectivey. The red arrows represent the perturbing forces exerted by the outer planet on the inner binary system.

amplitudes with a 1-minute precision (Agol & Steffen, 2007). So, if we could observe another solar system like ours with the transiting planet being Jupiter, with $a_t = 5.3$ AU, $P_t =$ 11 yr and $M_\star \approx 332,973 M_\oplus$ we can obtain possible values for a_c as a function of m_c for the detection of a transit time variation in this case. The result can be seen in Figure 3, which shows that a planet like earth could be detected if it were the only planet orbiting the central star. Although this result seems very encouraging, our current knowledge of detected exoplanets tells us that this is, today, very unlikely. Almost all of the jovian-like planets are at distances of ~ 0.5 AU from their parent star, so discovering a planet like ours in this case seems very unlikely for now.

2.2 Perturbations by an exterior planet on a large eccentric orbit

Consider an inner circular orbiting system formed by the central star and a transiting planet. If there's a third outer planet orbiting this inner binary system, the forces exerted by this planet on the inner binary perturbs the orbit, producing transit-timing variations.

Figure 4 makes clear that the forces exerted by the outer planet aren't constant because of it's high eccentricity. Borkovits et al. (2003) derive a general formula for this problem which involves a three body general system in terms of series expansions. On the other hand, Agol et al. (2005, hereafter A05) derives a first order Legendre series aproximation[2] for the perturbing effective force on the inner binary system using Jacobian coordinates (Murray & Dermott, 1999), which relate the position vectors of the components of the system from an arbitrary origin to the different centers of mass present in the problem. The TTV signal in this approximation is (see Agol et al., 2005, for the derivation):

$$\delta t = \beta \left(1 - e_o^2\right)^{-3/2} \left[f_o - n_o(t - \tau_o) + e_o \sin(f_o)\right]$$

Where the "o" subscript refers to the outer (perturbing) planet's properties and,

$$\beta = \frac{m_o}{2\pi(M_\star + m_t)} \frac{P_t^2}{P_o}$$

Where $M_\star$, m_o and m_t denote the masses of the star, the outer and the transiting planet and P_t and P_o denotes the transiting and outer planet periods. Note that $f_o(t)$ and the problem again reduces to solving the Kepler problem, but now for the outer planet considering that it orbits the center of mass. An approximation for the TTV signal rms value is given by A05 as:

$$< \delta t >_{rms} = \frac{3\beta e_o}{\sqrt{2}(1 - e_o^2)^{3/2}} \left[1 - \frac{3}{16} e_o^2 - \frac{47}{1296} e_o^4 - \frac{413}{27648} e_o^6\right]^{1/2}$$

Clearly the interesting case in this kind of perturbation are hot jupiters, but because the TTV signal scales as β, for the signal to give physical results we need $m_o \gtrsim m_t$ for hot jupiters (assuming a typical period of ~ 5 days, as proposed by Johnson et al., 2010).

2.3 Perturbations by general resonant and non-resonant planets on initially near-circular orbits

This case of perturbation is perhaps the easiest to visualize, but the hardest to control in analytical form. Figure 5 shows a perturbation by an inner planet on our outer (transiting) planet, where both started from circular orbits. For the case of non-resonant perturbations we can assume that the perturbation to the orbit of each planet is small, so the interactian can be calculated aproximately using the unperturbed orbits. Note that the conjunction period is $P_{conj} = 2\pi/(n_p - n_t)$ where n_p is the mean motion of the inner planet and n_t the mean motion of our transiting planet. Observe that initially these mean motions correspond almost exactly to the angular frequencies of the planets. This enable us to use linear perturbation theory (Hori, 1966, Deprit, 1968). Although the calculations of A05 enable us to derive interesting an encouraging results concerning the detection of earth-sized planets, the relatively recent work

[2] It is important to note that a Legendre series expansion for a central potential is natural because the generating function of the Legendre polinomials can in fact be derived from a central potential. As spherical coordinates are natural for spheres, Legendre series expansions are natural for central potentials.

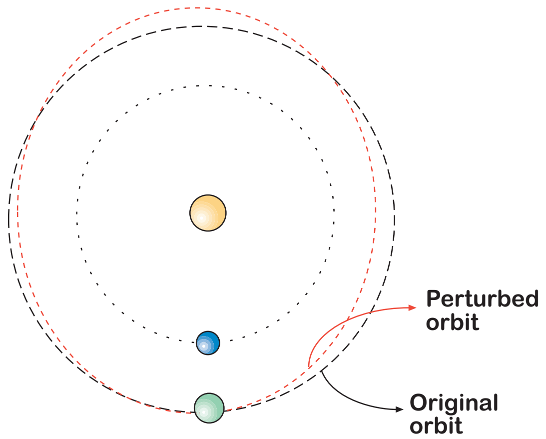

Figure 5. Illustration of the perturbation made by an inner planet on our outer planet by a strong perturbation at conjunction. Again, the yellow sphere represents the central star and the green and blue spheres represent the transiting and perturbing planets, respectivey. The red orbit indicates the perturbed orbit (note that the eccentricity now changes as a function of time).

by Nesvorný & Morbidelli (2008, hereafter NM08) provide a much general and modern approach.

Basically, NM08's perturbation theory-based iteration starts with a TTV signal and perturbs slightly the initial parameters. First, the algorithm obtains a measure of the mean period and starts perturbing this solution until it converges. In other words, this is a modern way of doing a N-body integration. Their simulation of a TTV signal simulated from N-bodies show a precision ranging from 2% to 7%. The greatest achievement of their work is the low CPU time requiered to perform the iterations. The exploration of the parameter space of 10^{10} planetary systems, for example, takes about a day of CPU time.

2.4 Satellite detections by transit timing variations

The detection of satellites orbiting extrasolar planets was first studied by Sartoretti & Schneider (1999). Basically, the detection relies on a combination of the same methods we analysed on previous sections, although careful work wasn't done on this subject until a recent theoretical work by Kipping (2009I, 2009II) who proposed the combination of the method of Transit Duration Variations (TDV, which is analog to the TTV but considering the duration of the transit), which consider the "in and outs" variations of the planet provided that a satellite is present on the exoplanet, therefore varying not only the time between transits but the duration of the transit itself. According to Kipping, the combination of the TTV and the TDV signals can provide several constaints on (probable) habitable exomoons.

3 RECENT SUGGESTIONS OF NEW LOW-MASS PLANETS USING TTV

Although up to now the confirmation of a planet by TTV is not widely accepted and usually needs radial velocity and/or transit follow ups, the recent suggestions of new low-mass planets using TTV signals of massive planets is an interesting field of reasearch on exoplanets. Here we sumarize two of the most recent suggestion made by the TTV method.

3.1 WASP-3b: Suggestion of a super-Earth

The recent study of Maciejewski et al. (2010) using TTV for the WASP-3 star suggests a $\sim 15M_{\oplus}$ planet that couldn't be detected earlier by radial velocity measurements. According to their conclusions, the assumption about a zero-eccentricity of WASP-3b biased this detection, because this assumption gives almost the same results as the proposed planet orbiting in a 2:1 resonance with WASP-3b. Using SYSTEMATIC-CONSOLE (Meschiari et al. 2009), they proved that leaving the eccentricity parameter free to vary and with the aid of their new data using TTV the chi-square value is reduced even further than when considering an eccentric orbit. Further follow up of this hypothesis using radial velocity measurements are needed to confirm the $\sim 15M_{\oplus}$ planet.

3.2 Kepler-9: The multiple system confirmed by TTV

Kepler-9 is a sun-like star (Holman et al., 2010) observed by the Kepler spacecraft with two recently discovered Saturn-size planets. The analysis of the variations on their periods, with TTV signals increasing and decreasing at averges of 4 and 39 minutes for each orbit, had recently identified an additional super-Earth-size planet candidate with a period of ~ 1.6 days. Further analysis will lead to more convincing conclusions about this subject.

4 CONCLUSIONS

Based on our investigation we could identify that Transit Timing Variations is a still growing field of reasearch on extrasolar planets. The complexities involved in the different methods of obtaining and analysing the transit-timing variation signal are really encouraging for further studies.

As we could see, transit timing variations could also be used to constrain the number of known planetary systems, supporting the study in almost every area of research in exoplanets: from habitable planet detection to statistics involving planets that could help to have a better understanding of planet formation.

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